Mathematics, an International Language

Mathematics. What does it mean? How is it taught at ISP? And how do our students learn it? Its roots are ancient, and yet it has endured as a discipline for millennia. It can be studied on its own, or as a cross-disciplinary field, spanning the sciences, art, architecture, psychology, computer science and music—just to name a few!

In this issue, you will be inspired by our dedicated and creative mathematics teachers, whose passion for the subject is felt within these pages. Discover also how mathematics has changed the lives of an ISP parent, our students and two of our alumni.

Whether you have math anxiety or think like Einstein, this issue of the Tribune underscores the accessibility of mathematics at ISP and helps us appreciate the power of its beauty and the way our teachers inspire students to conquer its complexities.
BEWARE, MATH IS LURKING!!

When I embarked on my higher education voyage in psychology and political science and then education, I never dreamed I would be as surrounded by figures as I am today. I never formally studied mathematics at university, apart from a few psychology-related statistics courses. However, despite the fact that I always enjoyed and succeeded at math, it was not my principal area of interest, and I never thought I would have to focus on it after university.

Now, as Head of School, I find myself immersed in admissions figures, budgets, financial forecasting, results, exchange rates, mortgage rates and staff appraisal ratings—and that’s just naming a few! This daily contact with numbers challenges me and allows me to be a lifelong learner in a field which is not my specialty.

Students, beware! No matter what profession you choose, mathematics is everywhere. So, pay attention in class, and see the numbers around you, in architecture, psychology, education, art, history, geography and of course all the sciences. You may just find that you start to enjoy it!

Math Talk: Interview with Elliott Rader, Grade 3

Sean Walker, PYP Coordinator

Elliott’s teacher, Sean Walker, interviewed him to find out what he thought about learning mathematics in the PYP.

What is mathematics?

How did mathematics work in Grade 2?
Really well! It was a lot of fun.

Why do we learn mathematics?
It’s fun! I use maths a lot when I need to know how much stuff there is. When I watch TV, I ask questions that have to do with maths. I also help my brother with money.

How is mathematics different here at ISP from your last school?
It’s in English! It’s easier to learn because it’s in English. I felt I didn’t learn anything before. Here I learn lots! I always loved maths signs and maths problems. There are some I don’t even know, but I like them. Square roots... plus signs...

What does a mathematician do?
Someone who works with maths and who works with numbers and shapes. They use a different language. They use different words. They use different names for corners of shapes like vertices... or edges for sides.

Interviewing Elliott, I couldn’t help but think about the Learner Profile, a set of 10 attributes all IB schools try to model and foster in the school community. At such a young age, Elliott was able to clearly communicate his love for mathematics and his genuine interest and curiosity in the subject. Already, he has become quite knowledgeable about the subject and has thought about the relevance of mathematics in his own life. Elliott is a young mathematician and demonstrates the qualities of the type of learner we strive to develop here at ISP.
Manipulating Mathematics in the PYP

Sean Walker, PYP Coordinator

In the PYP, students are encouraged to be mathematicians. Rather than simply memorize mathematical facts, they have regular opportunities to explore mathematical concepts, use manipulatives* to construct their own meaning, search for patterns and develop theories. Teachers provoke students’ thinking and encourage students to play and experiment with mathematical ideas while developing the language of mathematics and an increased use and awareness of mathematical notation. We aim to foster a love of learning mathematics in our students and develop their genuine interest in the subject.

At ISP, we also aim to develop children’s understanding of mathematical concepts. At the same time we recognize that there is an important set of skills and a body of knowledge that is necessary. This includes the recall of number bonds and multiplication facts that children throughout the grades are expected to learn by heart, after having had practical experiences to help them understand the operations of addition, subtraction, multiplication and division, as well as the relationships between them.

As mathematicians, students will be taught a variety of formal and informal strategies and procedures to solve a range of problems. Students are encouraged to explore and compare different approaches in an effort to find out what is most efficient and accurate for them. In each grade level, students are exposed to a broad range of mathematical ideas from numbers, shape and space, to measurement, pattern and function and data handling.

At times, students will inquire into these ideas in stand-alone mathematics lessons. However, when authentic connections are possible with other curriculum areas, they will learn other subjects through mathematics. For example, they might learn about a social studies concept by analyzing graphs, comparing distance of journeys by measuring, or describing the features of an art piece by identifying the names of shapes seen.

Students are continuously assessed in a variety of ways, sometimes by the teacher, sometimes by other students and sometimes by the student him/herself. Reporting on children’s progress in mathematics is extremely important here at ISP. This is often done informally through conversations between teachers and parents. However, there are also three written reports sent to parents each year that describe the progress made in mathematics. In addition, there are conferences scheduled throughout the year involving students, teachers and parents. This regular assessment allows teachers and students to reflect on how successful teaching and learning has been, to set future targets and to address misconceptions.

*Manipulatives are resources and hands-on materials that children can explore to explain and model their thinking.
Here’s a simple exercise. Think of a number, square it, then add the number you first thought of.

Now do the same, but keep repeating the process and recording the result so that you get a series of numbers. For some initial choices, the series will grow without limit; for others, it remains finite. This process is easy to analyze and not terribly exciting. But now, let’s allow the numbers to be complex. Don’t worry about the details—complex numbers are just ordered pairs of ordinary real numbers with suitably modified definitions of addition and multiplication. Again the series will either grow or remain bounded.

Since we are dealing with pairs of numbers, we can plot the results in a two-dimensional picture. Colors will represent numbers that grow, and black will represent the ones that do not. The result is a rather surprising shape, known as the **Mandelbrot set**, an object first described by Benoit Mandelbrot in 1980.

Now, let’s look at what happens near the border between the two regions. Near this boundary we have to do more repetitions before we can tell to which set the number belongs. Suppose further that we color-code each point depending on how many such iterations are needed. The results are some incredibly beautiful fractal images.

With the power of modern computers, we can probe the Mandelbrot set in great detail. As we inspect different areas, we find complex shapes and patterns that closely resemble things found in nature at all scales, from the microscopic to the cosmological.

In the adjacent pairs of images, each picture on the left is a photograph of some **naturally-occurring objects** and each picture on the right a mathematical pattern found deep within the Mandelbrot set.

We know that nature is complex, and there are many familiar ‘natural’ fractals: fern leaves, snowflakes and rugged coastlines, to name but a few. What seems to me to be so thought-provoking is that the deceptively simple mathematical recipe $z_{n+1} = z_n^2 + z_0$ somehow contains within it so many startling reflections of the real world.

There is indeed something very special about mathematics!
The Math Numeracy Diploma Club

Dr. Soraya Fathi, Mathematics & ToK Teacher, Secondary School

At the beginning of the last school year, the mathematics department seized the opportunity to increase the number skills of our grade 6 and 7 students. We created the Numeracy Diploma Club, which I run with the help and support of Mr. Ryan from the learning support department.

The club aims to provide our pupils with resources for delivering and assessing their number skills as well as a recording system for charting their progress and for recognizing and rewarding each student’s achievements. The diploma has three stages: recognizing Fundamental, Intermediate and Advanced numerical knowledge.

In each stage, students practice a set of skills, and each skill is resourced with practice sheets, tests and re-tests, all of which have answers. Students have a progress chart to record their achievements in the skills at their current stage. Once every skill at a stage has been achieved, a diploma is given to the students by Mr. Mansfield, Secondary School Principal, and a copy is kept in the student’s file.

Following the success and impact on learning and improvement in the numeracy skills of the students attending the club, we have decided to continue this activity this year and to expand it into grade 8.

We are excited about the progress our students have made and look forward to an even more stimulating year of learning and mathematical discovery.

Grade 9: Power Functions and Forensics

Stacy Chandler, MYP Coordinator

In Ms. Babulaud’s grade 9 extended mathematics class, students were asked to research careers that use math. It turned out that grade 9 students were passionate about forensics! Inspired by Agatha Christie novels, the students answered the question: “How would you determine the height of a suspect by measuring the diameter of blood splatters found on a crime scene?”

Using a special concoction made of raspberries meant to simulate blood, Ms. Babulaud’s students dropped drips from various heights and then conducted a series of investigations to look for patterns that could be linear, quadratic or power functions. Ultimately they figured out how to use the parabolic formula \( y = ax^2 + bx + c \). Such a learning exercise is called a “modeling activity.”

This exercise was in the spirit of open-ended inquiry. The results of learning are guided, but there is no single right answer. It provided the students with the recognition of how math permeates the world around us, the usefulness, power, and beauty of math and, finally, how to develop patience when solving problems.

“Last year I joined the Numeracy Diploma Club because I was having difficulties in math, and I thought it might help me. At the end of the year I looked back in my notebook, and my improvements were absolutely astonishing. I could also really see the improvement in my math class. In the Numeracy Diploma Club, we practiced multiplication, division, subtraction and addition. There are exercise sheets for three levels of difficulty; each time you complete a level you receive a diploma and move on to the next harder level. I was bad in division and this club was amazing in helping me with that problem. So if you have math difficulties and want to get better, I would really recommend this club, and you will be flabbergasted by your results at the end of the year.” —Dalya Soffer, Grade 7
Teaching Through Investigations

Mike Collett, Head of Mathematics Department, Secondary School

“When will we use this in our lives?” is a question that many students ask just as the teacher is explaining the use of the quadratic formula or some other algebraic concept. Using mathematical investigations is an important and integrated part of the math program at ISP in grades 6 to 12 and clearly demonstrates that math can be used as an everyday tool. There are three forms of investigation: the open-ended question, the use of mathematics to obtain a model that can be applied with a certain set of data, and finally, the everyday problem that can be solved using mathematics.

Knowledge gained in class enables students to answer many questions, but the open-ended investigation is a task that requires the student to derive a general formula from seeing a pattern in numbers or data. It is difficult for students to understand that often there is no one set answer and that the formulas they obtained may be different to someone else’s but are still correct.

All the tasks are graded under the four criteria of the MYP: knowledge and understanding (did the student use their knowledge and apply it to the task?); patterns; communication (is the student using words, symbols and graphs to problem-solve?); reflection (are the results reasonable, could other methods have been used?).

A major part of the IB diploma in mathematics is course work. All students in the high and standard level courses are required to produce a portfolio which contains two pieces of work, an open investigation and modeling question. The mathematical studies students produce a statistical investigation.

Mathematics has greatly evolved from traditional homework assignments based on solving problems in a textbook to a more dynamic and exciting student-centered approach.

Mathematical Problem Solving in the MYP

Dr. Soraya Fathi, Mathematics & ToK Teacher, Secondary School

Mathematics has always had two opposing aspects: skills and problem solving. Skills are seen as a set of well-defined procedures for transforming numbers, symbols or shapes; problem solving, in contrast, involves tackling tasks that are significantly different from those one has learned ‘by heart’.

If you look at mathematics as a set of procedural abilities, then the role of the student is to reproduce the procedure demonstrated by the teacher. Conversely, if mathematics is interpreted as an activity, which involves the implementation of a higher order set of abilities, a major part of the challenge lies in deciding how to tackle the problem, and which bits of one’s toolkit of mathematical skills will help.

A central question for the teaching and learning of mathematics in the Middle Years Programme is not which is more important. Rather, it is how these kinds of “knowing” interact, support or interfere with each other as students study mathematics. Offering mathematical ideas with rich interconnected procedural and conceptual features to students is essential to becoming mathematically proficient and for developing ‘procedural reasoning.’

In the MYP, problem solving is a means to enhance mathematical thinking so that teachers’ curricula documents, assessment procedures, and instructional practices provide students with the opportunity to become problem solvers. Engaging students with mathematical problem solving means providing them with a rich, connected understanding of mathematics and the ability to see patterns of similarity and association in the world around them.

The mathematics program at ISP not only provides students with the skills to carry out mathematical procedures but also enables them to plan how to attack a problem, and to check that the results make sense in the context of the problem.
Why Getting it Wrong is Right

Damian Kerr, Vice-Principal for Well-Being & Mathematics Teacher, Secondary School

There is a sense of instant gratification that students get in a mathematics class which is rarely found in other subjects. It is unusual, perhaps, for a student to feel that his history essay reached the perfection hoped for by the teacher; and the young scientist will be lucky to feel confident that her lab report on photosynthesis was perfect. Ask mathematics teachers, though, and they will all be able to tell you about relatively frequent examples of students high-fiving one another, or pumping the air, because they know for sure that what they have just done was exactly right. When students get it right, especially after persevering through the parts where they were not quite sure, they let you know, and it feels good.

But this is only half the tale. I tell my students that while getting it right is great, it is when they get things wrong that they are learning. What they find really difficult to get, though, is the idea that mistakes are valuable in themselves and that it is even more important to show their errors to the teacher. Many students try, get it wrong, and then erase what they have done. They then go to the teacher with a blank sheet, saying, "I can’t do it. Can you show me how?"

The better students (and this is the point) will try, get it wrong, and leave their work on the page, in all its mistaken glory. They then go to the teacher with a sheet full of unresolved problems, saying, "I can’t do it. Can you show me why?"

It might be going too far to say that this experience is even more gratifying than seeing students get it right the first time, but the interaction following the ‘why?’ is by far the most valuable one for me as a mathematics teacher. Showing students why they cannot do something is much more relevant to them than an explanation of how I can, and it almost inevitably leads to them having a better understanding of the work at hand.

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Grade 8 students, Emily, Mashide and Yuyang won second place at the Middle School Math Competition in Barcelona last April. Over 29 international schools were present. They are pictured here with mathematics teacher, Dr. Soraya Fathi, who congratulated them on their "commitment, academic ability, problem solving skills and team work."

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“I was never good at math”

Hugh Jackson, Mathematics Teacher, Secondary School

This is one of those phrases mathematics teachers hear occasionally at three-way conferences, which usually precedes the explanation of how the child equally has trouble coping with the work in the classroom. It tends to presume that mathematics is somehow hereditary, and that the child has started off at a disadvantage.

Although there may exist some argument of nature against nurture, it would seem strange if we applied the same argument to driving a car, for example. You do not need to be a rocket scientist to develop the skills needed to drive a car but you do need time, patience and practice. It is the same for speaking a second language; children become fluent without too much effort if exposed regularly at an early age.

Mathematical ability is developed in the same way. It is as if we are climbing a wall where different topics may rise up side-by-side, as we develop further concepts which lift us higher. As Newton said, “If I have seen further it is only by standing on the shoulders of giants”. Hopefully we can dissuade our students from attempting similar acrobatic feats in the classroom, but reflecting on what their strengths and weaknesses are can help them fill in some of the gaps in their wall of knowledge.
Theory of Knowledge (ToK) is designed to give grade 11–12 Diploma students an opportunity to reflect on what it means to know something and on how they change as their knowledge changes. The course tries to define what knowledge and knowing are; it moves into different ways of knowing and areas of knowledge, and it also looks at knowledge and values—especially ethical values and decisions that we all face.

Within the Theory of Knowledge course, mathematics is discussed as an area of knowledge on its own. The purpose of the module is not to learn new mathematical concepts. Rather, students focus on what mathematics is, and on how mathematical knowledge differs from other areas of our life.

Central questions to the module are: What is mathematics? How do you know that various mathematical statements are true? Is mathematics driven by cultural assumptions and beliefs? Does mathematics describe reality? Is mathematics a language?

Students address these questions by exploring topics of discussion such as Fermat’s last theorem, the notion of mathematical proof, geometry as an axiomatic system, and mathematical deduction versus experimental induction. Certainly exploring mathematics from a ToK perspective provides students with the opportunity to enrich and broaden their view on the subject by reflecting critically on what it means to think mathematically.

Sophie is currently enjoying the challenges of high-level math with Mr. Kerr. She stopped by our office one day to share some insights into why math is one of her favorite subjects:

**What has been your history with mathematics?**
I have always loved math. When I was in middle school, however, I had a math teacher who was very exacting and intimidating. We were all afraid of him! However, when I entered high school, I had a series of excellent math teachers who made the subject enjoyable again.

**What changed in high school?**
I used to get stuck on a problem and just stop working on it. What I learned from these good teachers was to try a different approach, a different path to finding the right answer. Math became fun again!

**How would you help a student who is having trouble with math?**
I would say to watch out for those trigonometry identities; there are a lot of questions that seem simple at first, but then you end up going around in circles. There are tricks you can learn to help you understand. When it works—like with vectors and 3-D planes—it is a really great feeling!

**Beyond good teachers, what has helped you feel comfortable with math?**
I tutored young students in math. Teaching helped me learn a lot, not only in reviewing what I had already learned, but knowing that I could help kids really understand and get it.

**Any final thoughts?**
I’m not in high-level math just to try to get a 7 on the IB diploma exam; I am in this class because I truly enjoy what the IB Math course teaches. We cover so many different topics, and we do so with lots of depth. Because I am also interested in English literature and history, I am applying to liberal arts colleges where I will be able to continue exploring different fields. Another reason I love math is that it also appeals to my more creative side; there are lots of different ways to get to the right answer. You learn a lot more by taking these less-obvious paths where there is more room for creativity. So, who knows? I might be a mathematician or a poet!

“I’ve Always Loved Math!”
An interview with Sophie Diamond, Grade 12

Theory of Knowledge and Mathematics
Dr. Soraya Fathi, Mathematics & ToK Teacher, Secondary School

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Mathematics in Time & Space

Lillian Hueber, Mathematics Teacher, Secondary School

Mathematics is often understood as the closest thing we have to an international language devoid of national prejudice and independent of the century we find ourselves in. It aims to capture, understand and convey what lies behind the observable world by stripping away the subjectivity of the observer. Hence, the story goes, all mathematics could be explained and understood by anyone, regardless of where or when the person lived or that person’s native language.

Pupils learn mathematics that was discovered (or invented—another argument for another article) somewhere between our hunter-gatherer years and this last century’s computer scientists. The idea of counting and of numbers has been with humans for a very long time. The Ishango Bone, thought to be a primitive calculator or counting aid, dates from about 20 000 years ago and was found in the area between the modern-day Democratic Republic of the Congo and Uganda.

Children develop the idea of numbers early on. Graph theory, on the other hand, is a topic studied in the MYP and is being researched today due to its relevance to, for instance, computer networking and the study of chemical molecules. Generally speaking, the older pupils are, the more abstract the mathematics taught and the later in human history it was developed. As such, learning mathematics sometimes resembles a trip down humanity’s memory lane.

Just as mathematics spans millennia with not too much trouble, it also has an international breadth that is difficult to fit into the school curriculum. What is taught as Pythagoras’ Theorem was discovered several times, not only by Egyptians before the Greek philosopher, but also in BCE China where today it is known as Gougu Dingli. As history was written, some names were emphasized and the contribution of others was lost, sometimes remembered only in nomenclature. Algebra and algorithm, for instance, originated from the title of a book by the 9th century Persian scholar in Baghdad, Abū Abdallāh Muhammad ibn Mūsā al-Khwārizmī.

One challenge to the mathematics teacher is to teach the examined curriculum and convey this historical and cultural wealth, as appreciating this aspect of the subject might make mathematics more accessible and interesting to many students.

“Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.”

David Hilbert (1862-1943)
As you settle back into school after an exciting summer, mathematics might be one of the last things on your mind. Does it really matter if you can solve a quadratic equation or not? It seems just like yesterday that I was sitting in classes at ISP, but for me mathematics (and the way it was taught at ISP) made all the difference to where I am now.

I can proudly say that mathematics at ISP was fun. I was lucky enough to have two great teachers (Mr. Kerr & Mr. Stansfield) who not only made the subject both appealing and interesting but who also paid a great deal of attention to my personal development. The foundation that I developed in mathematics set me on the path toward understanding how and why things work, which has been invaluable in my academic and professional life.

I started out my academic life in physics, which is typically very math intensive, and ended up working with computer networks, which most people think is math intensive—but is not. I think that’s what makes math unique; it has an application, great or small, to perhaps everything we do. Very few people are pure mathematicians; I can’t think of anyone who doesn’t have some need for math in their everyday life. I have little use for the chemistry and Latin I learned in school, but while I have probably forgotten most of my math of physics days, I have to exercise the rest every day.

Some might argue that math and language skills are the universals (they certainly are when it comes to testing). But languages vary in their syntax, and some say in the meanings that are expressed, whereas I don’t see anyone arguing that division is useless or should be replaced with some new mathematical operator when you move to a new country.

So I think that’s what is special about math: it’s everywhere, and the same everywhere. Maybe that’s a clue—math is really the statement of all things that are everywhere and the same everywhere, and we get to add what’s different as we apply it.
“From now on, you will sleep with your graphical calculator, and you will become best friends!” Those were Mr. Kerr’s welcoming words to my first IB mathematics class at ISP in 2005. A few years later, I am so attached to my calculator that I am now incapable of figuring out what 18/3 is without it.

I have always enjoyed numbers, but I never saw myself pursuing a profession consisting solely of mathematics. ToK classes, however, gave me a new perspective on the world of math. While writing my ToK essay, I realized we cannot solve mathematical problems only through reasoning; we also need our imagination to help create this mathematical world in our minds. The fascinating aspect of mathematics is that it is not a secluded field in itself but critical to many other fields of study and to different sciences.

My extended essay in Environmental Systems awakened another area of passion in my life. The ISP groundwork in both math and the environment sent me off to where I am currently studying: a BSc in BioMathematics in my home country, South Africa (University of Stellenbosch). This is a newly-developed course based on mathematical modeling within ecology.

Along with the global vision I acquired from ISP’s outstanding diversity, I am optimistic that I will become a part of a small group of young African scientists in 2011 studying at the African Institute of Mathematical Sciences close to Cape Town. There, I will continue with a postgraduate degree in BioMathematics, discovering the world of mathematics in our surrounding environment.

ISP inspired me into a profession. I will always be grateful for the way ISP shaped my thinking processes and how the School and its staff helped me discover my abilities and my potential.

Marinél Janse van Rensburg, Class of 2007

“Mathematics is like draughts [checkers] in being suitable for the young, not too difficult, amusing, and without peril to the state.”

Plato (429-347 BCE), Plato’s Academy
Can you solve this puzzle?

The 25 letters A to Z (omitting o) represent the 25 prime numbers less than 100 in some order. The usual mathematical notation is used, and no answer starts with a zero.

Can you solve the puzzle and find the number represented by "ISP"?

<table>
<thead>
<tr>
<th>ACROSS</th>
<th>DOWN</th>
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<tbody>
<tr>
<td>1. t²</td>
<td>1. eq</td>
</tr>
<tr>
<td>4. q² - (d² + h²)</td>
<td>2. l² + d² - m</td>
</tr>
<tr>
<td>7. c³</td>
<td>3. kl²</td>
</tr>
<tr>
<td>8. p² - z²</td>
<td>4. z² - (c + v)</td>
</tr>
<tr>
<td>10. mj²</td>
<td>5. a(v - s)</td>
</tr>
<tr>
<td>12. l</td>
<td>6. n² + g² - x</td>
</tr>
<tr>
<td>13. sw</td>
<td>9. w³</td>
</tr>
<tr>
<td>15. u</td>
<td>11. ajy</td>
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<tr>
<td>16. b</td>
<td>14. k(q + y)</td>
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<td>17. f² + k²</td>
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<td>18. t</td>
<td>19. gx</td>
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<td>19. m(b + w)</td>
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<tr>
<td>21. s</td>
<td>22. p² - v²</td>
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<tr>
<td>23. r² + u²</td>
<td>24. my - j</td>
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<tr>
<td>25. a! - fh</td>
<td>26. cd + c</td>
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<td>27. u² + mu³ - tu</td>
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<tr>
<td>28. in</td>
<td></td>
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<tr>
<td>29. (i + l)(p + r)</td>
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</table>

Check your puzzle: send answers (or questions) to reception@isparis.edu.

“So far as the theories of mathematics are about reality, they are not certain; so far as they are certain, they are not about reality.” Albert Einstein (1879-1955)